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The eigensolutions of a two-dimensional Kemmer oscillator

Abdelmalek Boumali

Laboratoire de Physique Théorique et Appliquée L P T A, Université de Tébessa, 12000 W. Tebessa, Algeria

E-mail: aboumali@yahoo.fr and aboumali@mail.univ-tebessa.dz

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Abstract

Two-dimensional eigenfunctions and eigenvalues of massive spin-1 particles in the presence of the Dirac oscillator have been found by using the Kemmer equation. We derive a complete analytical solution of the system, describing in detail the energy spectrum and associated eigensolutions by using the formalism of the chiral creation–annihilation operators.

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1. Introduction

In relativistic quantum mechanics, exact solutions of the wavefunction are very important for understanding the physics that can be obtained by such solutions. The Dirac-like Kemmer equation is not new and dates back to the 1930s (for historical details, see [1]). Historically, the loss of interest in the Kemmer equation stems from the equivalence of the Kemmer approach to the Klein–Gordon (KG) and Proca descriptions in on-shell situations, in addition to the greater algebraic complexity of the Kemmer formulation. However, in the 1970s this supposed equivalence began to be investigated in several situations involving the breaking of symmetries and hadronic process, showing that in some cases Kemmer and KG theories can give different results. Thus, one important question concerning the Kemmer equation is whether there is equivalence or not between its spin-0 and -1 sectors and the theories based on the second-order KG and Proca equations, respectively [2]. Moreover, the Kemmer equation seems to be richer than the KG one with respect to the introduction of interactions. In this context, alternative Kemmer-based models were proposed for the study of meson–nucleus interactions, yielding a better adjustment to the experimental data when compared to the KG-based theory [3]. In the same direction, approximation techniques formerly developed in the context of nucleon–nucleus scattering were generalized, giving a good description of the experimental data of meson–nucleus scattering [4]. The deuteron's nucleus scattering was also studied using the Kemmer equation, motivated by the fact that this theory suggests a spin-1 structure by combining two spin-$\frac{1}{2}$ [5]. In the same context, we can cite the works of Barrett and Nedjadi [6] and Ait-Tahar *et al* [7] on the meson–nuclear interaction and the relativistic model of α-nucleus elastic scattering where they have been treated by the formalism of the Kemmer theory. In the last decade, we have noted a renewed interest in the Kemmer equation. It has been studied in the context of QCD [8], covariant Hamiltonian [9], in the causal approach [10, 11], in the context of five-dimensional Galilean invariance [12], in the scattering of K^+ nucleus [13], in the presence of the Aharonov–Bohm potential [14, 15], in the Dirac oscillator interaction [16], in the study of thermodynamics properties [17], in the presence of some shape of interactions [18–29], etc. These examples, among others in the literature, in some cases break the equivalence between the theories based on Kemmer and KG and Proca equations, such as in [3] or in Riemann spacetimes [30].

The Dirac oscillator (DO) is one of the most important quantum systems, as it is one of the very few that can be solved exactly. It was studied by Itô *et al* [32] for the first time. On the other hand, Moshinsky and Szczepaniak [33] were the first who introduced an interesting term in the Dirac equation. More specifically, they suggested substituting in the free Dirac equation the momentum operator $\vec{p}$ such as $\vec{p} - \mathrm{i}m\beta\omega\vec{r}$, with $\vec{r} = (x, y, z)$ being the position vector, m the mass of the particle and ω the frequency of the oscillator. They could obtain a system in which the positive energy states have a spectrum similar to that of the non-relativistic harmonic oscillator. It can be shown that the Dirac oscillator interaction is a physical system, which can be interpreted as the interaction of the anomalous magnetic moment with a linear electric field [34, 35]. The Dirac oscillator has aroused a lot of interest both because it provides one of the few examples of the exact solvability of the Dirac equation and because of its numerous physical applications. As a relativistic quantum mechanical problem, the DO has been studied from many viewpoints, including covariance properties, complete energy spectrum and corresponding wavefunctions, symmetry Lie algebra, shift operators, hidden supersymmetry, conformal invariance properties as well as completeness of wavefunctions. Relativistic many-body problems with Dirac oscillator interactions have been extensively studied with special emphasis on the mass spectra of mesons (quark–antiquark systems) and baryons (three-quark systems). The dynamics of wave packets in a Dirac oscillator has been determined and a relation with the Jaynes–Cummings model established. (2 + 1) spacetime has also been shown to be an interesting framework for discussing the DO in connection with new phenomena (such as the quantum Hall effect and fractional statistics) in condensed matter physics. The thermodynamic properties of the DO in (1+1) spacetime have been mentioned to be relevant to studies on quark–gluon plasma models [36].

In this work, we want to derive a complete solution of the two-dimensional Kemmer oscillator, using the formalism of the chiral creation and annihilation operators. We show that the Kemmer equation can be solved exactly, and the energy spectrum and the corresponding wavefunction have been obtained. This paper is organized as follows. In section 2, we review the solutions of the two-dimensional Dirac oscillator. In section 3, the solutions of the two-dimensional Kemmer oscillator by using the formalism of the chiral creation–annihilation operators have been sought. Section 4 is devoted to the discussion of different results obtained. Finally, section 5 presents the conclusion.

2. Eigensolutions of the two-dimensional Dirac oscillator

The free Dirac equation in (2+1) spacetime is

$$[c\overrightarrow{\alpha}\cdot\overrightarrow{p} + \hat{\beta}mc^2]\psi = E\psi, \tag{1}$$

where ψ is the two-component Dirac spinor, $\vec{\alpha}$ are known as the standard Dirac matrices which can be expressed in terms of the Pauli matrices $\vec{\sigma}$, $\vec{p}$ is the momentum operator and c stands for the speed of light. When we introduce the Dirac oscillator interaction in the free Dirac equation, equation (1) becomes

$$[c\overrightarrow{\alpha}\cdot\left(\overrightarrow{p}-\mathrm{i}m\omega\beta\vec{r}\right)+\hat{\beta}mc^2]\psi_{\mathrm{D}}=E\psi_{\mathrm{D}}. \tag{2}$$

In the case of a two-dimensional problem, the Dirac matrices become 2×2 matrices, which can be identified with the so-called Pauli matrices as follows:

$$\hat{\alpha}_x=\hat{\sigma}_x=\begin{pmatrix}0&1\\1&0\end{pmatrix},\qquad \hat{\alpha}_y=\hat{\sigma}_y=\begin{pmatrix}0&-\mathrm{i}\\\mathrm{i}&0\end{pmatrix},\qquad \hat{\beta}=\hat{\sigma}_z=\begin{pmatrix}1&0\\0&-1\end{pmatrix}. \tag{3}$$

In this form, ψ can be described by a two-component spinor $\psi_{\mathrm{D}}=(\psi_1\quad\psi_2)^T$, and equation (2) transforms into

$$[c\alpha_x\cdot(p_x-\mathrm{i}m\omega\beta x)+c\alpha_y\cdot(p_y-\mathrm{i}m\omega\beta y)+\hat{\beta}mc^2]\psi_{\mathrm{D}}=E\psi_{\mathrm{D}}. \tag{4}$$

This equation was algebraically solved by introducing chiral creation and annihilation operators [37–39]:

$$a_r=\tfrac{1}{\sqrt{2}}(a_x-\mathrm{i}a_y),\qquad a_r^\dagger=\tfrac{1}{\sqrt{2}}\left(a_x^\dagger+\mathrm{i}a_y^\dagger\right), \tag{5}$$

$$a_l=\tfrac{1}{\sqrt{2}}(a_x+\mathrm{i}a_y),\qquad a_l^\dagger=\tfrac{1}{\sqrt{2}}\left(a_x^\dagger-\mathrm{i}a_y^\dagger\right), \tag{6}$$

where a_x, $a_x^\dagger$, a_y, $a_y^\dagger$ are the usual annihilation and creation operators of the harmonic oscillator

$$a_i^\dagger=\frac{1}{\sqrt{2}}\left(\frac{r^i}{\triangle}-\mathrm{i}\frac{\triangle}{\hbar}p^i\right),\qquad i=(x,y), \tag{7}$$

with $\Delta=\sqrt{\frac{\hbar}{m\omega}}$ representing the ground oscillator width. The orbital angular momentum may be expressed as

$$L_z=\hbar\left(a_r^\dagger a_r-a_l^\dagger a_l\right), \tag{8}$$

which leads to a physical interpretation of $a_l^\dagger$ and $a_r^\dagger$: these operators create a left or right quantum of angular momentum, respectively, and are hence known as circular creation–annihilation operators. These operators allow an insightful derivation of the energy spectrum [37]

$$E_\pm=\pm mc^2\sqrt{1+4\zeta n_l}, \tag{9}$$

where the integer n_l stands for the number of left-handed orbital quanta, and $\zeta=\frac{\hbar\omega}{mc^2}$ is an important parameter that specifies the importance of relativistic effects in the Dirac oscillator. The corresponding total wavefunction for both positive and negative eigenstates has the following form [37]:

$$|\pm E_{n_l}\rangle=\begin{bmatrix}\sqrt{\frac{E_{n_l}\pm mc^2}{2E_{n_l}}}\,|n_l\rangle\\ \mp\mathrm{i}\sqrt{\frac{E_{n_l}\mp mc^2}{2E_{n_l}}}\,|n_l-1\rangle\end{bmatrix}. \tag{10}$$

In the non-relativistic limit, where $\zeta\to 0$, we obtain

$$E_\pm=\pm mc^2(1+2\zeta n_l). \tag{11}$$

In this manner, we have exposed the two-dimensional Dirac oscillator describing the energy spectrum and the eigenstates in terms of chiral quanta. As follows, we concentrate on the case of massive spin-1 particles confined in the low-dimensional Dirac oscillator interaction and governed by the Kemmer equation.

3. Eigensolutions of the two-dimensional Kemmer oscillator

The free relativistic Kemmer equation is [40–42]

$$(\beta^{\mu} p_{\mu} - Mc)\psi_K = 0, \tag{12}$$

where M is the total mass of two identical spin-$\frac{1}{2}$ particles. β matrices, called Kemmer matrices, are 16×16 matrices and have three irreducible representations of dimensions 1, 5 and 10. They satisfy the following commutation relation:

$$\beta^{\mu}\beta^{\nu}\beta^{\lambda} + \beta^{\lambda}\beta^{\nu}\beta^{\mu} = g^{\mu\nu}\beta^{\lambda} + g^{\lambda\nu}\beta^{\mu}, \tag{13}$$

with

$$\beta^{\mu} = \gamma^{\mu} \otimes \hat{1} + \hat{1} \otimes \gamma^{\mu}, \tag{14}$$

obeying the commutation rules first introduced by Duffin [41]; $\hat{1}$ is a 4×4 identity matrix, γ^{μ} are the Dirac matrices and $\otimes$ indicates a direct product [43]. The stationary state ψ_K of equation (12) is a four-component wavefunction of the Kemmer equation, which can be written in the form

$$\psi_K = \psi_{\mathrm{D}} \otimes \psi_{\mathrm{D}} = (\psi_1 \quad \psi_2 \quad \psi_3 \quad \psi_4)^T, \tag{15}$$

where ψ_{D} is the solution of the Dirac equation. In the presence of the Dirac oscillator potential, the momentum operator $\vec{p}$, in the free Kemmer equation, could be substituted by $\vec{p} - \mathrm{i}MB\omega\vec{r}$, where the operator B in the additional term is chosen as $B = \gamma^0 \otimes \gamma^0$, with $B^2 = \hat{1}$. In this case, the Kemmer equation with a Dirac oscillator interaction is

$$[(\gamma^0 \otimes I + I \otimes \gamma^0)E - c(\gamma^0 \otimes \vec{\alpha} + \vec{\alpha} \otimes \gamma^0) \cdot (\vec{p} - \mathrm{i}M\omega B\vec{r}) - Mc^2\gamma^0 \otimes \gamma^0]\psi_K = 0. \tag{16}$$

At this stage, three remarks, which seem important to us, can be made as follows [16]. First, the massive spin-1 particle, which we consider here, constitutes a two-particle system of spin-$\frac{1}{2}$ instead of a single spin-1 particle [18], and therefore the Kemmer equation is a two-body Dirac-like equation. Second, the β-matrices in equation (14) are in their reducible representation, and each of the three inequivalent representations of β^{μ} is contained just once in the particular β^{μ} representation of equation (14). Third, the wavefunction ψ_K is assumed to be a product of two Dirac wavefunctions ψ_{D}, and the operator B is chosen as a direct product of two γ^0 operators, instead of η^0 used in [18]. For these reasons, although we used the same equation which described the spin-0 and spin-1 particles, the two formalisms do not give the same results (see [16, 18]).

Now, in the case of a 2D Dirac oscillator, the standard Dirac γ matrices are replaced by Pauli σ matrices, and consequently equation (16) becomes

$$[(\gamma^0 \otimes I + I \otimes \gamma^0)E - \sqcap - \sqcup - Mc^2\gamma^0 \otimes \gamma^0]\psi_K = 0, \tag{17}$$

where

$$\sqcap \equiv c(\gamma^0 \otimes \sigma_x + \sigma_x \otimes \gamma^0) \cdot (p_x - \mathrm{i}M\omega Bx), \tag{18}$$

$$\sqcup \equiv c(\gamma^0 \otimes \sigma_y + \sigma_y \otimes \gamma^0) \cdot (p_y - \mathrm{i}M\omega By). \tag{19}$$

Putting equation (15) into equation (17), we easily obtain four linear algebraic equations:

$$(2E - Mc^2)\psi_1 - c\{(p_x + \mathrm{i}M\omega x) - \mathrm{i}(p_y + \mathrm{i}M\omega y)\}\psi_2 \\ - c\{(p_x + \mathrm{i}M\omega x) - \mathrm{i}(p_y + \mathrm{i}M\omega y)\}\psi_3 = 0, \tag{20}$$

$$Mc^2\psi_2 - c\{(p_x - \mathrm{i}M\omega x) + \mathrm{i}(p_y - \mathrm{i}M\omega y)\}\psi_1 \\ + c\{(p_x - \mathrm{i}M\omega x) - \mathrm{i}(p_y - \mathrm{i}M\omega y)\}\psi_4 = 0, \tag{21}$$

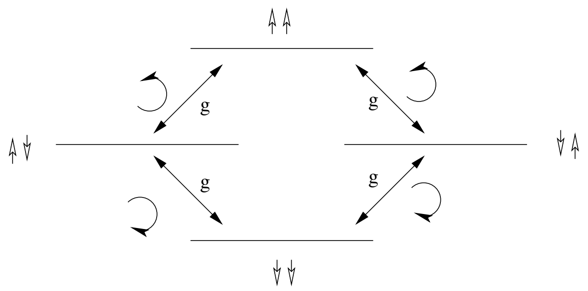

Figure 1. Relativistic coupling scheme for different levels.

$$Mc^2\psi_3 - c\{(p_x - \mathrm{i}M\omega x) + \mathrm{i}(p_y - \mathrm{i}M\omega y)\}\psi_1 \\ + c\{(p_x - \mathrm{i}M\omega x) - \mathrm{i}(p_y - \mathrm{i}M\omega y)\}\psi_4 = 0, \tag{22}$$

$$-(2E + Mc^2)\psi_4 + c\{(p_x + \mathrm{i}M\omega x) + \mathrm{i}(p_y + \mathrm{i}M\omega y)\}\psi_2 \\ + c\{(p_x + \mathrm{i}M\omega x) + \mathrm{i}(p_y + \mathrm{i}M\omega y)\}\psi_3 = 0. \tag{23}$$

From these equations, we find

$$\psi_2 = \psi_3, \tag{24}$$

$$\psi_1 = \frac{2c}{2E - Mc^2}\{(p_x + \mathrm{i}M\omega x) - \mathrm{i}(p_y + \mathrm{i}M\omega y)\}\psi_2, \tag{25}$$

$$\psi_4 = \frac{2c}{2E + Mc^2}\{(p_x + \mathrm{i}M\omega x) + \mathrm{i}(p_y + \mathrm{i}M\omega y)\}\psi_2. \tag{26}$$

Like in the case of a two-dimensional Dirac oscillator of particles of spin -$\frac{1}{2}$

$$p_x = \frac{\mathrm{i}M\omega\Delta}{\sqrt{2}}\left(a_x^\dagger - a_x\right), \qquad x = \frac{\Delta}{\sqrt{2}}\left(a_x^\dagger + a_x\right), \tag{27}$$

$$(p_x + \mathrm{i}M\omega x) - \mathrm{i}(p_y + \mathrm{i}M\omega y) = g a_l^\dagger, \tag{28}$$

$$(p_x + \mathrm{i}M\omega x) + \mathrm{i}(p_y + \mathrm{i}M\omega y) = g a_r^\dagger. \tag{29}$$

Equations (25) and (26) transform into

$$\psi_1 = \frac{2g}{2E - Mc^2} a_l^\dagger \psi_2, \tag{30}$$

$$\psi_4 = \frac{2g}{2E + Mc^2} a_r^\dagger \psi_2, \tag{31}$$

with $g = 2\mathrm{i}Mc^2\sqrt{\zeta}$ being the coupling constant between the different spinor components, and ζ is a parameter that controls the non-relativistic limit.

The relativistic coupling scheme is depicted in figure 1. From this figure, one can note that the components ψ_1 and ψ_4 can be obtained from the component ψ_2 via the chiral creation–annihilation operators. By considering the two-body spinorial basis $\{|\uparrow\uparrow\rangle, |\uparrow\downarrow\rangle, |\downarrow\uparrow\rangle, |\downarrow\downarrow\rangle\}$, both transitions between the different spinorial components and the fermionics spin-flip transitions are mediated through the creation or annihilation of chiral quanta [39]. Thus, this situation represented in figure 1 is very similar to the well-known fundamental model that describes the interaction of the two-level atom with the quantized mode of the electromagnetic field. Thus, one can note that this figure shows a correspondence with the relativistic Jaynes–Cumming (JC) model in quantum optics [45].

Now, from equation (27), we find

$$(p_x - \mathrm{i}M\omega x) + \mathrm{i}(p_y - \mathrm{i}M\omega y) = -ga_l, \tag{32}$$

$$(p_x - \mathrm{i}M\omega x) - \mathrm{i}(p_y - \mathrm{i}M\omega y) = -ga_r. \tag{33}$$

In this case, the above system of equations becomes

$$\begin{bmatrix} 2E - Mc^2 & -ga_l^\dagger & -ga_l^\dagger & 0 \\ -ga_l & Mc^2 & 0 & ga_r \\ -ga_l & 0 & Mc^2 & ga_r \\ 0 & ga_r^\dagger & ga_r^\dagger & -(2E + Mc^2) \end{bmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = 0_{4\times 1}. \tag{34}$$

Here, three remarks are essential as follows.

- First, a particular combination of the chiral operators is demanded by the conservation of the total angular momentum:

$$J_z = L_z + S_z, \tag{35}$$

  where we have introduced the z-component of the total spin and angular momentum operators:

$$L_z = \hbar\left(a_r^\dagger a_r - a_l^\dagger a_l\right), \qquad S_z = \sigma_z \otimes I_2 + I_1 \otimes \sigma_z. \tag{36}$$

- Second, we can remark that the form of equation (34) is similar to that of the relativistic JC coupling in the quantum optical model. Equation (34) contains a couple of spin-$\frac{1}{2}$ particles coupled to a pair of chiral modes, like in the case of a quantum optical model involving a pair of two-level atoms coupled to two modes of an electromagnetic field [39, 45, 46].
- Third, according to equation (34), it seems impossible to deduce directly the form of the Hamiltonian in order to diagonalize it. Nonetheless, it is possible to find the complete energy spectrum and the corresponding eigenstates.

Now, when we write the component ψ_2 in terms of the left and right chiral quanta bases, i.e $\psi_2 \equiv |n_l, n_r\rangle$, with

$$|n_l, n_r\rangle = \frac{1}{\sqrt{n_l n_r}}\left(a_l^\dagger\right)^{n_l}\left(a_r^\dagger\right)^{n_r}|0\rangle, \qquad [a, a^\dagger] = 1, \tag{37}$$

we obtain

$$\left[Mc^2 + \frac{2cg^2}{2E - Mc^2}\left(1 + a_l^\dagger a_l\right) - \frac{2cg^2}{2E + Mc^2}\left(1 + a_r^\dagger a_r\right)\right]|n_l, n_r\rangle = 0. \tag{38}$$

After a simple algebraic calculation, we get the following equation of eigenvalues:

$$E^2 + 4Mc^2\zeta(n_l - n_r)E - 2(Mc^2)^2\zeta(2 + n_l + n_r) - \left(\frac{M}{2}c^2\right)^2 = 0. \tag{39}$$

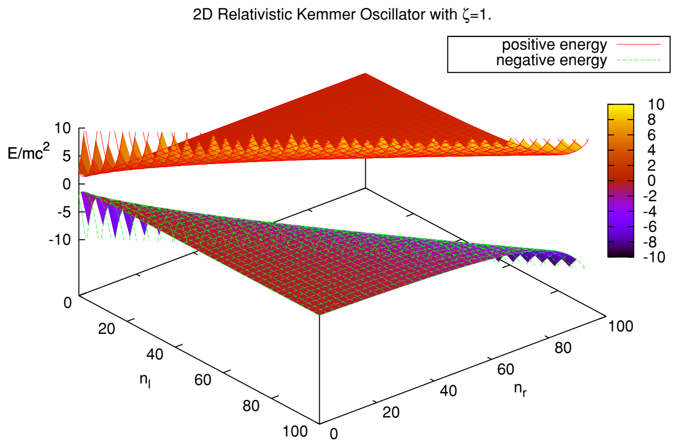

Figure 2. Relative energy spectrum E/Mc^2 as a function of the chiral quanta numbers (n_l, n_r) with $\zeta = 1$.

(This figure is in colour only in the electronic version)

Finally, the eigensolutions of the 2D Kemmer oscillator are

$$E_{\pm} = Mc^2 \left\{ 2\zeta\,(n_l - n_r) \pm \sqrt{4\zeta^2\,(n_l - n_r)^2 + 2\zeta\,(2 + n_l + n_r) + \frac{1}{4}} \right\}, \tag{40}$$

$$\left(\psi_{n_l,n_r}\right)_K = \begin{bmatrix} \frac{2g}{2E-Mc^2}\sqrt{n_l+1}|n_l+1, n_r\rangle \\ |n_l, n_r\rangle \\ |n_l, n_r\rangle \\ \frac{2g}{2E+Mc^2}\sqrt{n_l+1}|n_l, n_r+1\rangle \end{bmatrix}. \tag{41}$$

In the non-relativistic limit, where $\zeta \to 0$, equation (40) becomes

$$E_{\pm} = Mc^2\left(\pm\tfrac{1}{2} \pm \zeta + 2\zeta[n_l - n_r] \pm 2\zeta[n_l + n_r]\right) \tag{42}$$

or

$$E_{+} = Mc^2\left(\tfrac{1}{2} + \zeta + 4\zeta n_l\right), \tag{43}$$

$$E_{-} = -Mc^2\left(\tfrac{1}{2} + \zeta + 4\zeta n_r\right). \tag{44}$$

4. Results and discussion

The formalism of chiral creation and annihilation operators applied in the case of the 2D Kemmer oscillator gives an exact form of the energy of the spectrum for massive spin-1 particles. In figure 2, we show the dependence of both positive and negative energies with three parameters, ζ, n_l and n_r; there is no interference in the figure and, consequently, the well-known Zitterbewegung phenomena disappear. This effect is a term describing the jittery movements of a particle due to interference between parts of its wave packet belonging to positive and negative energy states [47]. In figure 3, we have plotted the energy levels as a function of the coupling parameters ζ for four levels. We can see that when the chiral numbers are equal, $n_l = n_r$, the energy levels are simple, well separated but

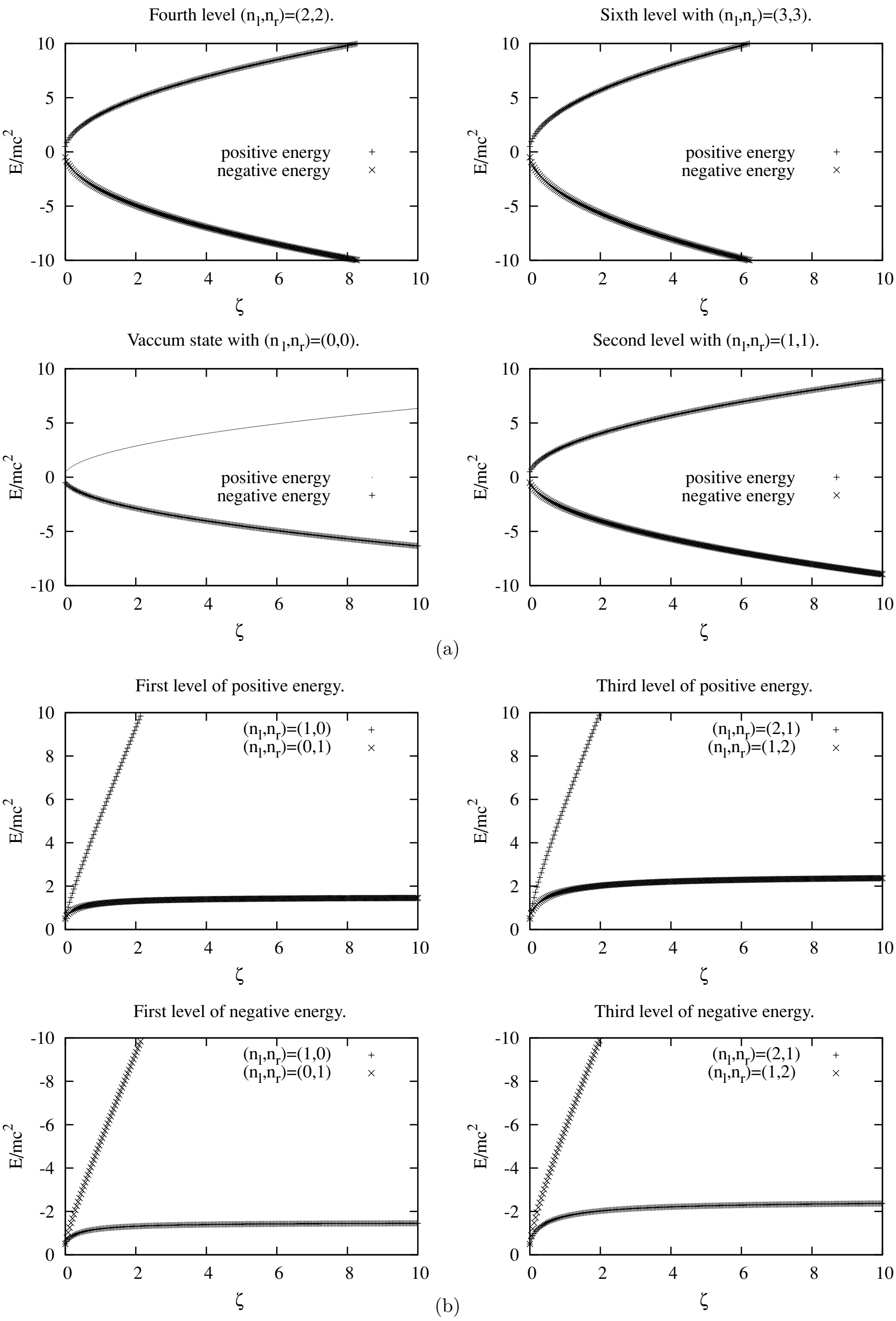

Figure 3. Relative energy spectrum E/Mc^2 as a function of ζ. (*a*) Both positive and negative energy for levels where $n_l = n_r$. (*b*) Both positive and negative energy for levels where $n_l \neq n_r$.

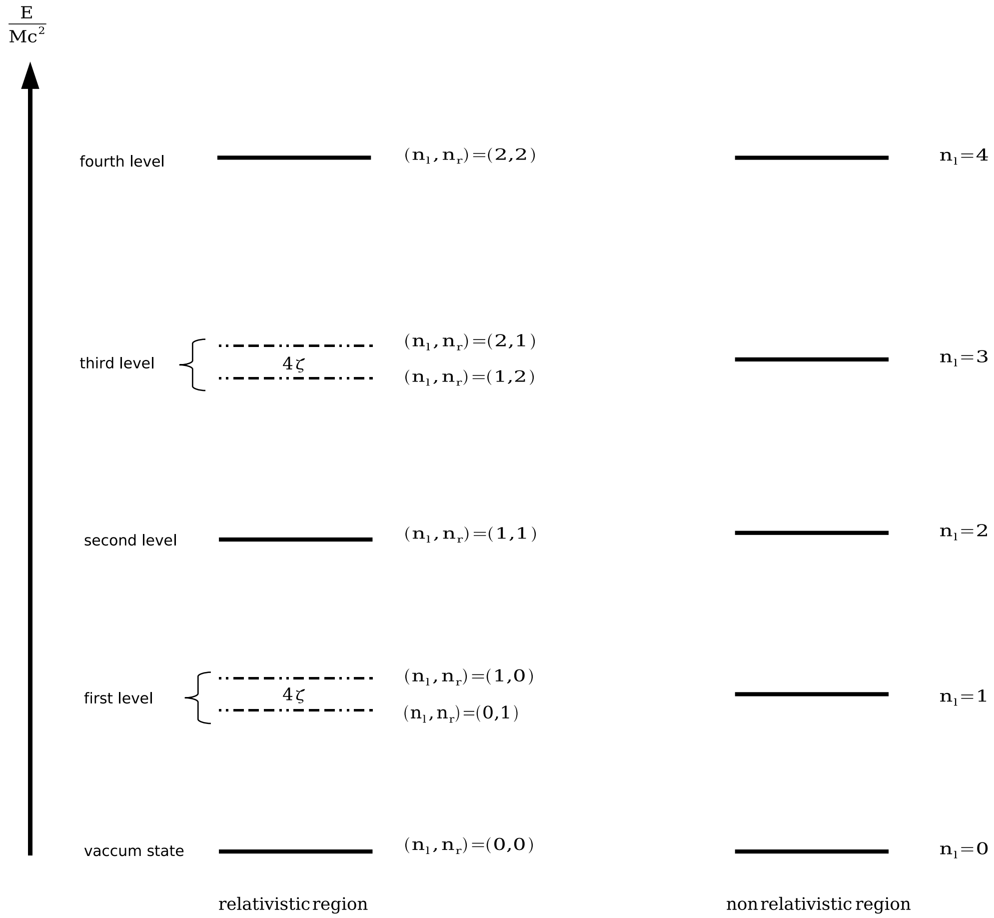

Figure 4. The diagram of the energy of the 2D relativistic Kemmer oscillator for five levels.

not equally spaced, and the appropriate level is $|\uparrow\uparrow\rangle\,|n_l+1, n_r\rangle$ or $|\downarrow\downarrow\rangle\,|n_l, n_r+1\rangle$. But when the chiral numbers are different, $n_l \neq n_r$, all levels of energies are degenerated and separated by an interval of 4ζ. These observations can be explained and argued like in the case of a two-body system [39] as follows: both $|\uparrow\downarrow\rangle\,|n_l, n_r\rangle\,|\uparrow\downarrow; n_l, n_r\rangle$ and $|\downarrow\uparrow\rangle\,|n_l, n_r\rangle$ are the same states and, consequently, we have two possible spin-flip channels of opposed chirality. Namely, $|\uparrow\downarrow\rangle\,|n_l\rangle \leftrightarrow |\uparrow\uparrow\rangle\,|n_l+1\rangle \leftrightarrow |\downarrow\uparrow\rangle\,|n_l+1\rangle$ is a right-handed channel, whilst $|\uparrow\downarrow\rangle\,|n_r\rangle \leftrightarrow |\downarrow\downarrow\rangle\,|n_r+1\rangle \leftrightarrow |\downarrow\uparrow\rangle\,|n_r\rangle$. Both channels conserve the total angular momentum. The different situations shown in figure 3 have been depicted as an energetic diagram for both relativistic and non-relativistic regions according to the ζ parameter in figure 4; according to this figure, the impair order shows a degeneracy of these levels in contrast to the case of pair order which is simple and no degeneracies exist. In the non-relativistic region, all levels become single.

Now, we discuss the results found here compared with those obtained in the literature and, precisely, we concentrate on the study of the connection between the Kemmer oscillator and the model of the relativistic two-body Dirac oscillator. Rozmej and Arvieu [48] showed that the Dirac oscillator interaction is a relativistic version of the JC model, and proved that when using the Foldy–Wouthuysen (FW) transformation, the Zitterbewegung phenomena disappears. Benitez *et al* [49] found that the exact solution in three dimensions has a hidden supersymmetry responsible for the special degeneracies of its energy spectrum. The connection between the two-body Dirac equation and the Kemmer equation has been the object of the several studies that one can mention the works of Aydin [50]. In an interesting paper by Kaelberman [51], the two-body Dirac equation was reduced to a one-body Kemmer equation in the case of equal masses. In the presence of the Dirac oscillator interaction, Bednar *et al* [52] studied the connection between the two-body Dirac and Kemmer equations in three dimensions. They used the results obtained from the works of Nedjadi and Barrett [18], based on the DKP equation, and the works of Moshinsky *et al* [53] using the two-body Dirac formalism. They showed that all these approaches admit hidden parasupersymmetry and point out the reducibility of the two-body Dirac oscillator. Recently, Bermudez and Martin-Delgado [54] have studied the confinement of two fermions in the Dirac oscillator potential, in (1+2) dimension, and have obtained a complete analytical solution. In order to compare our study to that of [54], one notes the following. (i) The problem we treated here is a problem at two dimensions in contrast to that in [52] which is a problem at three dimensions. Recently, the 2D problems have become very important in physics in both theoretical and experimental aspects, and as an example we can cite the graphen system which is a 2D crystal where the particles are confined and obey the Dirac equation. (ii) To the best of our knowledge, it seems impossible to deduce directly the form of the Hamiltonian in our case, and consequently the comparison with the Hamiltonian model based on the two-body Dirac oscillator will not be direct. In spite of all, one can make the following remarks.

- Both theories are reducible and do not mix $S = 1$ and $S = 0$ states.
- Both studies show a good correspondence with the well-known Jaynes–Cumming model in quantum optics.
- The two-body Dirac oscillator and the Kemmer oscillator, in this particular case, give a complete analytical solution; the difference sums up on the complexity of the eigenvalues in [54] in relation to our energy spectrum. Perhaps the existence of a hidden parasupersymmetry can be the reason for this disagreement.

Finally, as mentioned in section 1, the thermal properties of the DO in (1+1) spacetime have been mentioned to be relevant to studies on quark–gluon plasma models. Unfortunately, and in spite of the great number of papers that have been recently published concerning the solutions and properties of the Kemmer equation, as far as we know, no one has reported on its thermal properties with the exception of the (1+1) dimensions case [17]. Pacheco *et al* [55] showed that the Dirac oscillator can be used for describing the confinement of quarks in mesons and baryons. On the other hand, Nedjadi and Barrett [18] suggested that the DKP oscillator could also be a good candidate to be used as the confinement potential in heavy quark potential. Thus, according to these results, we can hope that the eigensolutions of the 2D Kemmer oscillator can be used to calculate the thermodynamics properties of the Kemmer oscillator at (1+2) dimensions.

5. Conclusion

The Kemmer equation in the presence of the Dirac oscillator interaction is exactly solved. In the relativistic case, we cannot see any interference between positive and negative energies, and so the well-known Zitterbewegung phenomena disappear. From the scheme of the diagram of energies, two cases are presented: the case where $n_l = n_r$ gives simple, and no degenerate, levels and the case where $n_l \neq n_r$ gives the degeneracies of these levels. Finally, we can expect that the model of confinement of two fermions by the Dirac oscillator potential at two dimensions, based on the Kemmer equation, can be employed as a good tool to obtain the well-known models of Jaynes–Cumming (JC) in quantum optics. As an extension of this work, we intend to carry out studies on the thermal properties of the Kemmer oscillator in two dimensions.

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